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ESTIMATE OF THE PHYSICAL EXTENT OF THERMAL PLUMES AT LAKEVIEW T.G.S. 1971

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ESTIMATE OF THE PHYSICAL

EXTENT OF THERMAL PLUMES

AT LAKEVIEW

ABSTRACT

The cooling water plumes from an operating thermal electric generating station at Lakeview, west of Toronto, on Lake Ontario, were defined by water temperature measurements. The generating station discharged between 1400 to 2500 cfs of cooling water at 15°F above intake water temperature. The temperature measurements were made throughout the year at various depths in proximity of the discharge to define the physical cooling water plume in the lake. A method for estimating temperature isotherms was developed by statistically analyzing the temperature measurements. Equations were developed which defined plume area as a function of distance from the discharge, temperature as a function of plume area and plume centreline as a function of wind. Different relationships were developed for summer and winter conditions based upon the ambient lake temperature. The most significant cooling resulted from mixing with lake water.

ESTIMATE OF THE PHYSICAL EXTENT OF THERMAL PLUMES AT LAKEVIEW

INTRODUCTION

With the advent of large thermal electrical generating stations employing Great Lakes water for cooling, there has been increasing concern on the possible detrimental effects from the addition of heat to the water environment. Research on the effect of heat inputs on the water environment has been extensive *(Parker, 1969 and ASCE, 1967)*. However, the methods for predicting the physical extent of cooling water plumes on lakes are notably few. This aspect is considered essential by the Ontario Water Resources Commission. Once defined, the local effects of the heated plume on the water environment could be examined.

Studies were undertaken by the Hydro-Electric Power Commission of Ontario (H.E.P.C.) to define the physical extent of the cooling water plume at the Lakeview generating station on Lake Ontario. Lakeview discharged approximately 1400-2500 cfs at 15°F above intake water temperature. Initially, the studies consisted of measuring water temperatures with a telethermometer at predetermined locations in proximity of the discharge throughout the year. Normally, a survey took less than 3 hours. Meteorological data collected during the surveys consisted of wind and air temperature determinations.

This report discusses the development of a first order estimate of the spatial distribution of isotherms within the cooling water plume. The estimate is a first order approximation based on limited survey data. A better estimate could be made by solving the heat budget equation. However, the solution of the heat budget equation requires more extensive measurements of wind, relative humidity, air and water temperatures, net solar radiation and water movement patterns. Measurements of these parameters are now being made by H.E.P.C.

Csanady (1970) recently showed that rate of heat lost to atmosphere is one-third of that lost to the lake in studies at Douglas Point on Lake Huron. *Palmer (1969)* also showed that excess heat in a plume is dissipated through horizontal mixing with the main lake. It will be shown that the principal cooling mechanism appears to be mixing with the ambient lake water. These results are in conflict with *Brady (1969) and Parker (1969)* who both indicate that the main cooling is atmospheric. *Riesbol (1971)* in studies for a cooling water discharge on Dardanelle Lake in Arkansas found that cooling by entrainment of lake water is the main mechanism in the first several thousand feet while atmospheric cooling prevails farther away from the discharge point.

EXPERIMENTAL METHOD

Water temperature surveys were conducted by H.E.P.C. with a small fast boat on relatively calm days. Surface

and some depth temperatures were measured at the intersection points of a 1000 feet square geometric grid. The intersection points were defined by visual alignment on shore range poles with some assistance from shore based optical transits when required. All temperatures were measured with a Yellow Springs thermister telethermometer having an accuracy of 0.2° F.

An average water temperature survey was completed in less than three hours. During the period of survey, the following parameters other than water temperature were measured several times on shore and then averaged.

1. Air temperature in °F
2. Wind speed in mph
3. Wind direction in degrees from north
4. Rate of discharge from outfall in cfs

Observed isotherms were plotted for each survey. The centreline of the plume was drawn by joining the farthest points of each isotherm from the outfall. The area enclosed by each isotherm was measured by planimeter.

PRELIMINARY INVESTIGATIONS

Several different methods for predicting lake temperature isotherms from the available data were tried with varying degrees of success in terms of matching actual observed isotherms. Some of the methods tried are described briefly.

Gifford's Normal Distribution Dispersion Model

This method assumes that the concentration distribution of any continuously discharged material in a moving stream will be normally distributed across any plane perpendicular to the moving stream. As the form of the concentration distribution is defined, i.e. a normal distribution, it is only necessary to define spread (variance) and current as functions of downstream location to determine concentrations at different distances from the source. This method has been applied successfully in meteorology *(Gifford, 1955)*, ocean discharges *(Foxworthy, 1966)* and lakeshore areas *(Palmer, 1969)*.

$$\bar{C}_{max}(x) = \frac{2Q}{\pi U \sigma^2\{o\}\left(\frac{2\sigma^2\{x\}}{\sigma^2\{o\}} + 1\right)} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

where

$\bar{C}_{max}(x)$ = maximum mean concentration at distance x from source in lbs/ft^3

Q = continuous mass flow in lbs/sec

U = current velocity in ft/sec

$\sigma^2\{o\}$ = variance of concentration distribution at discharge in ft^2

$\sigma^2\{x\}$ = variance of concentration distribution at distance x from discharge point in ft^2

This method was applied with some success to a cooling water plume in May 1968 *(Palmer, 1969)*. The method considers dilution with ambient lake water. Since lake cooling has been shown as the principal cooling mechanism at least in the first few thousand feet, this method could be used effectively

provided current velocities are known. Current velocities in near shore areas with shore topographical effects, could not be satisfactorily predicted from wind velocities. Therefore, the method was rejected.

Johns Hopkin's Method

This method is based on statistical fitting of the principle mechanisms responsible for cooling in small ponds or lakes *(Brady, 1969)*. The technique was successfully applied to experimentally determined plumes with the computed parameters comparing favourably with other studies. However, it was necessary to define the portion of the lake affected by the plume to apply the method. For small lakes, this is simply the total surface area. As it was not possible to satisfactorily define the area of the lake affected, this method was abandoned.

PROPOSED METHOD

The above methods described briefly could not be used successfully with the available survey data. Therefore, a statistical approach was used to develop a prediction model for the thermal plume. The data used is listed in Appendix 2.

The field survey data was divided into two seasons, i.e. winter and summer since the entire data set produced statistically weaker equations with large standard errors of estimates. The equations were improved by separating the data into 'winter' and 'summer' sets. Since much of cooling is due to mixing with lake water and air temperatures are very

variable, ambient lake water temperature was chosen as the criterion for separation of data. Lake temperatures of 42°F, 45°F and 47°F were tried for separation of data. 45°F was found to be the best since it produced statistically best prediction equations for both sets. Best fit equations were developed for the following:

a. area (A) contained by isotherms and the distance (X) (the farthest point on isotherm from the outfall, measured along the plume centreline)

b. area (A) and temperature difference (ΔT_{PL}) between isotherm corresponding to area and ambient lake temperature

c. distance (X) and temperature difference (ΔT_{PL})

From the first two relations, it is possible to define plume area and temperature at any distance (X) from the outfall. These equations are necessary to determine the plume temperature and area at any distance from outfall to enable plotting of the plume. The relation between X and ΔT_{PL} has been obtained for completeness, but cannot be used to determine isotherm shapes since it does not define isotherm area. Various shapes, namely circular, elliptical and one leaf of multi-leaved rose were investigated as possible shapes for the predicted isotherm. One leaf of multi-leaved rose was found to approach the observed isotherm shape best and was adopted in the present model. The direction of the plume centreline was determined from the wind vector. This method will be described in the

section 'Definition of Plume Centreline'.

Plume Area and Distance

Relations have been found for area or spread or variance as a function of distance from a source by many experimenters *(Alsafar, 1966; Csanady, 1964; Okubo, 1968; Foxworthy, 1966 and Murthy, 1969)*. In the case of near-shore areas of the Great Lakes, the relations appear to be general in slope for a particular location, but vary in intercept for the maximum, mean and minimum conditions *(Palmer and Izatt, 1970)*. On this basis, it is probable that a statistically significant equation could be determined relating area (contained by the isotherms) of the plume and distance from outfall along the plume centre-line. The best fitted equation (see Table 1 and Figure 2) for the summer data set was:

$$\text{Area} = -0.696 + 1.384\ (\text{Distance}) \qquad (2)$$

with $F_{1,95} = 117.1$: multiple correlation coefficient = 0.74: standard error = 3.11: standard deviation area = 4.62

The best winter data equation (see Figure 3) was:

$$\text{Area} = -4.047 + 2.43\ (\text{Distance}) \qquad (3)$$

with $F_{1,63} = 283.6$: multiple correlation coefficient = 0.90: standard error = 2.49: standard deviation area = 5.80.

It is immediately apparent that there was a statistically significant relationship between the areas contained by the

isotherms and the distance from the outfall. The winter equation was statistically stronger, when lake temperature variations were smallest.

Isotherm Area and Temperature Difference (Plume and Ambient Lake Temperature)

The cooling of the plume to either the atmosphere or the lake must be related to the plume area. The entrainment of lake water at ambient temperature is in part a function of the plume area in contact with the lake water. Similarly, the amount of atmospheric cooling is in part a function of the surface area. If one assumes that the plume area in contact with the atmosphere is nearly the same as that in contact with the lake, one would expect a relationship to exist between the area contained by an isotherm and the temperature difference between plume and ambient lake temperature. This seems to be a valid assumption as most of the thermal plume appears to be confined to the surface regions of the lake. A statistically significant relationship was found (see Table 3 and Figure 4). The best equation for the summer data including a flow term was:

$$\text{Area} = 90.53 - 8.86 \ln (\text{flow X temperature difference}) \quad (4)$$

with $F_{1,95} = 135.2$: multiple correlation coefficient = 0.77: standard error = 2.99: standard deviation area = 4.62.

The best winter equation including a flow term (see Figure 5) was:

Area = 74.0 - 6.88 ln (flow X temperature difference)................(5)

with $F_{1,63}$ = 126.3: multiple correlation coefficient = 0.82: standard error = 3.37: standard deviation area = 5.80.

The question immediately arises whether a better equation could be determined if temperature differences between the plume and atmosphere as well as the temperature difference between ambient lake and plume were included. Statistical fitting of the summer and winter data including both temperature differences is tabulated in Table 2. Significant equations were found relating the temperature differences and isotherm area and for the summer data the standard error was slightly less. However "t" tests on the significance of the two temperature differences indicate that the air-plume difference was not significant for the summer data and not significant at α= 0.01 for the winter data. The predominate cooling mechanism in these surveys was lake water cooling. As the method proposed here is a first order estimate, it was decided for simplicity to ignore atmospheric cooling and utilize the relationship between isotherm and lake-plume temperature difference.

<u>Isotherm Distance and Temperature Difference</u>

(plume and ambient lake)

Although this relationship was not required to predict the isotherms, it is useful to obtain rapid estimates of maximum distances isotherms will occur away from the discharge point, without plotting actual isotherms. The equations are summarized in Table 4. It is observed that the equations were not as statistically significant as the other fits and the standard errors over standard deviation of distance were larger than the other fits. The best summer equation including a flow term is:

$$\text{Distance} = 33.08 - 3.0 \ln (\text{flow X temperature difference}) \quad \ldots\ldots\ldots\ldots\ldots (6)$$

with $F_{1,95} = 29.0$: multiple correlation coefficient = 0.48: standard error = 2.18: standard deviation of distance = 2.48.

The best winter equation including a flow term was:

$$\text{Distance} = 26.88 - 2.30 \ln (\text{flow X temperature difference}) \quad \ldots\ldots\ldots\ldots\ldots (7)$$

with $F_{1,63} = 73.4$: multiple correlation coefficient = 0.73: standard error = 1.48: standard deviation = 2.16.

The winter equation is the significantly better prediction equation (Figure 6).

Definition of Plume Centreline

The curvature or radius of the plume was defined by fitting the measured radius of the plume centreline and the absolute value of the resolved component of the wind acting parallel to the shoreline. The locii for the centres for

the plume radii were determined by trial-and-error and appear in Figure 7. The generality of these locii in terms of other locations is not known particularly as there was an effect on the plume from the cooling water intake channel configuration. The best fit relationship for radii and wind component appears in Figure 8.

$$R = 500|Wp| \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where R = plume centreline radius in ft.

Wp = wind component along the shore in mph.

It is apparent that the point scatter is large for equation (8). The standard error of estimate can be improved by defining a unique locus of the centre of the radius for each experiment. However, this is of little value for a technique of representing the thermal plume. Other methods involving current measurements could not be used as the measurements are incomplete.

<u>Step Description of Method</u>

From the meteorological data, for the period under consideration, the wind component along the shoreline was computed and thence the radius of plume centreline from the equation (8). The plume centreline is drawn according to the definition of Figure 7.

For the existing lake temperature, falling in 'summer' or 'winter' regimes, the appropriate equations were selected from Tables 1 and 3. Plume temperatures and areas were computed at various distances from outfall and along the plume centreline for a particular discharge. For any distance, the area computed represented the plume contained

by the corresponding temperature isotherm. One leaf of the multiple-leaved rose was then fitted to the distance and plume area. A step by step computation appears in Appendix I.

Three examples of the method are presented in Figures 9,10 and 11 for lake temperatures of 33°F, 46.5°F and 71.7°F and wind vectors acting in three different directions. The actual field survey results appear as solid line isotherms whereas the predicted plumes are broken lines. The shaded area for the plume predictions represent one standard error of estimate for the plume area with isotherm enclosing the smaller area representing the mean value. Similarly, the temperature range is one standard error of estimate with the upper value representing the mean value. Errors of locating the plume centreline have not been included and the centreline represents the mean value location.

DISCUSSION

The isotherm predictions in Figures 9 to 11 do approximate the measured isotherms. The fact that the predictions are first order estimates with large standard errors is immediately apparent. Statistical fitting of data which knowingly do not encompass all the determining mechanisms, e.g. currents, thermal conductivity of lake, radiation, etc. has many shortcomings. In some cases, it would have been possible to estimate some of the mechanisms like currents and radiation from other surveys or semi-empirical equations. However, these estimations were

avoided as experience has indicated that currents very near to shore and near the surface tend to be very local in nature. Particular caution is also indicated in the Lakeview area where major shoreline modifications like the cooling water intake and wharf structure have taken place.

The method is a first order estimate based on statistical fitting of limited data taken at one operating thermal generating station. It is suggested that the errors of estimate should be presented with the estimates to clearly identify the limitations of the methods. Transposing the predictions to another area has not been tested.

CONCLUSIONS

A method for predicting the isotherms of a surface cooling water discharge was developed statistically from temperature surveys of an existing thermal plume on Lake Ontario. However, the errors of estimating the plume were large indicating that a more comprehensive measuring program is needed to improve the predictions. The measuring program should be continuous over many hours and include measurements of all the important heat exchange mechanisms. The heat lost by mixing with colder lake water was significantly greater than the heat lost to the atmosphere (not considering evaporation implicitly).

REFERENCES

Alsaffar, A.M., Lateral diffusion in a tidal estuary, J. Geophy. Res., V.71, pp. 5837, 1966.

ASCE, Bibliography on Thermal Pollution, Proc. Am.Soc. Civ.Engs., V.96, No. SA3, pp. 85-113, 1967.

Brady, D.K., W.L. Graves, Jr. and J. C. Geyer, Cooling Water Studies for Edison Electric Inst., The Johns Hopkins Univ., Res. Proj. RP-49, EEI, Publication No. 69-901, 750 Third Ave., New York, N.Y. 10017.

Csanady, G.T., Hydrodynamic studies on Lake Huron at Baie du Dore, Summer 1964, Water Resources Inst., PR.19, University of Waterloo, Waterloo (Ont.), Canada, 1964.

Csanady, G.T., W.R. Crawford and B. Pade, Thermal Plume Study at Douglas Point Lake Huron, Environmental Fluid Mechanics Laboratory, University of Waterloo, Waterloo (Ont.) 1970.

Foxworthy, J.E., R.B. Tibby and G.M. Barsom, Dispersion of a surface waste field on the sea, J. Water Pollution Control Federation, V.38, No. 7, pp. 1170-1193, 1966.

Murthy, C.R., Large scale diffusion studies at Niagara River mouth Lake Ontario. Proc. 12th Conf. Great Lakes Res., p.635, 1969.

Okubo, A., A new set of oceanic diffusion diagrams, Chesapeake Bay Inst., The Johns Hopkins Univ., Tech. Rept. 38, Ref. 68-6, 1968. Distributed by: Clearinghouse for Federal Scientific and Tech. Inf. # AD675269, Springfield, Va. 22151 (U.S.A.)

Palmer, M.D. and J.B. Izatt, Lakeshore Two-Dimensional Dispersion, Ontario Water Resources Commission, 135 St. Clair Ave. W., Toronto 195, (Ont.), Canada, 1970.

Palmer, M.D., Simulated Thermal Effluent into Lake Ontario, Proc. 12th Conf. Great Lakes Res., p.674, 1969.

Parker, F.L. and P.A. Krenkel, Thermal Pollution: Status of the Art, Rept. No. 3, Dept. Envir. Water Resources Eng., Vanderbilt Univ., Nashville, Tennessee (U.S.A.), 1969.

Riesbol, H.S., J.B. Anderson, F.H. Wend and H.T. Holmes, Thermal-hydraulic study: Arkansas cooling reservoir, Proc. Am. Soc. Civ. Engs., v.97, No. PO1, p.93, 1971.

TABLE 1

Statistical Fitting

Plume Area (A) and Distance from Outfall (X)

Data Set	Equation	Std. Dev. A	F	F @ $\alpha=0.01$	Multiple Correlation	Std. Error
Summer[1]	$A^{\#} = -0.696 + 1.384X^{*}$	4.62	$F_{1,95}=117.1$	6.91	0.74	3.11
	$A = 2.11 + 0.124X^2$	4.62	$F_{1,95}= 96.2$	6.91	0.71	3.28
	$A = 4.38 + 0.00029e^X$	4.62	$F_{1,95}= 28.5$	6.91	0.48	4.07
Winter[2]	$A = -4.047 + 2.43x$	5.80	$F_{1,63}=283.6$	7.06	0.90	2.49
	$A = 0.91 + 0.239x^2$	5.80	$F_{1,63}=283.3$	7.06	0.90	2.49
	$A = 4.31 + 0.0033e^x$	5.80	$F_{1,63}=114.0$	7.06	0.80	3.48

Notes: # A (Area) in 10^6 $ft.^2$

* X (Distance) in 10^{-3} ft.

[1] 'Summer' is defined when lake temperature $\geq 45°F$

[2] 'Winter' is defined when lake temperature $< 45°F$

TABLE 2

Statistical Fitting

Plume Area (A*) vs Temperature Differences $\Delta T_{PL}^{*\#}$ (Plume-Lake) and $\Delta T_{PA}^{\#*}$ (Plume-Air)

Data Set	Equation	Std. Dev. A	F	F @ α=0.01	Multiple Correlation	Std. Error
'Summer'[1]	$A = 105.16 - 10.83 \ln (Q^{\#}\Delta T_{PL}) + 0.53 \ln (Q\Delta T_{PA})$ $Q\Delta T_{PL}$: $t = 10.65$; $t_{0.01}=2.384$ $Q\Delta T_{PA}$: $t = 1.03$; $t_{0.01}=2.384$	4.676	$F_{2,72}=72.7$	4.91	0.82	2.73
'Winter'[2]	$A = 76.77 - 8.45 \ln (Q\Delta T_{PL}) + 1.25 \ln (Q\Delta T_{PA})$ $Q\Delta T_{PL}$: $t = 8.98$; $t_{0.01}=2.395$ $Q\Delta T_{PA}$: $t = 2.17$; $t_{0.01}=2.395$	5.194	$F_{2,57}=42.6$	5.00	0.77	3.35

NOTES:

* A (Plume Area) x 10^6 ft^2.

\# Q (Discharge from outfall) in cfs

*# ΔT_{PL} (Temperature difference between plume and lake) $^{\circ}F$

#* ΔT_{PA} (Temperature difference between plume and air) $^{\circ}F$

[1] 'Summer' is defined when lake temperature $\geq 45^{\circ}F$

[2] 'Winter' is defined when lake temperature $<45^{\circ}F$

TABLE 3

Statistical Fitting

Plume Area (A) and ($Q\Delta T_{PL}$)

Data Set	Equation	Std. Dev. A	F	F @ $\alpha = 0.01$	Multiple Correlation	Std. Error
'Summer'[1]	$A^{\#} = 14.56 - 1.06\ (\Delta T_{PL}^{\#*})$	4.62	$F_{1,95}=112.4$	6.91	0.74	3.14
	$A = 14.14 - 0.00055\ (Q^{*}\Delta T_{PL})$	4.62	$F_{1,95}=98.4$	6.91	0.71	3.26
	$A = 23.67 - 8.79 \ln (\Delta T_{PL})$	4.62	$F_{1,95}=142.5$	6.91	0.77	2.94
	$A = 90.53 - 8.86 \ln (Q\Delta T_{PL})$	4.62	$F_{1,95}=135.2$	6.91	0.77	2.99
	$A = 23.53 - 6.28 \sqrt{\Delta T}_{PL}$	4.62	$F_{1,95}=130.0$	6.91	0.76	3.02
	$A = 23.25 - 0.14 \sqrt{Q\Delta T}_{PL}$	4.62	$F_{1,95}=118.9$	6.91	0.75	3.10
'Winter'[2]	$A = 15.91 - 0.89\ (\Delta T_{PL})$	5.80	$F_{1,63}=102.9$	7.06	0.79	3.60
	$A = 13.97 - 0.00034\ (Q\Delta T_{PL})$	5.80	$F_{1,63}=59.8$	7.06	0.70	4.19
	$A = 23.70 - 7.81 \ln (\Delta T_{PL})$	5.80	$F_{1,63}=176.8$	7.06	0.86	2.99
	$A = 74.00 - 6.88 \ln (Q\Delta T_{PL})$	5.80	$F_{1,63}=126.3$	7.06	0.82	3.37
	$A = 24.76 - 5.81 \sqrt{\Delta T}_{PL}$	5.80	$F_{1,63}=155.4$	7.06	0.84	3.14
	$A = 21.75 - 0.11 \sqrt{Q\Delta T}_{PL}$	5.80	$F_{1,63}=94.3$	7.06	0.77	3.70

Notes:
- # A (Area) in $10^6\ ft^2$
- * Q (Discharge) in cfs
- #* ΔT_{PL} (Temperature difference between plume and lake) in °F
- 1 'Summer' is defined when lake temperature $\geq 45°F$
- 2 'Winter' is defined when lake temperature $< 45°F$

<u>TABLE 4</u>

Statistical Fitting

Plume Distance $(X)^{\#}$ and Temperature Difference $(\Delta T_{PL})^{*}$

Data Set	Equation	Std. Dev. X	F	F @ α= 0.01	Multiple Correlation	Std. Error
'Summer'[1]	$X = 7.49 - 0.38\ (\Delta T_{PL})$	2.48	$F_{1,95}$=29.8	6.91	0.49	2.17
	$X = 7.39 - 0.00020(Q^{\#} * \Delta T_{PL})$	2.48	$F_{1,95}$=28.5	6.91	0.48	2.19
	$X = 10.25 - 2.9\ \ln\ (\Delta T_{PL})$	2.48	$F_{1,95}$=27.9	6.91	0.48	2.19
	$X = 33.08 - 3.0\ \ln\ (Q\Delta T_{PL})$	2.48	$F_{1,95}$=29.0	6.91	0.48	2.18
	$X = 10.49 - 2.16\ \sqrt{\Delta T}_{PL}$	2.48	$F_{1,95}$=29.5	6.91	0.49	2.18
	$X = 10.48 - 0.05\ \sqrt{Q\Delta T}_{PL}$	2.48	$F_{1,95}$=29.5	6.91	0.49	2.18
'Winter'[2]	$X = 7.73 - 0.32\ (\Delta T_{PL})$	2.16	$F_{1,63}$= 87.8	7.06	0.76	1.40
	$X = 7.00 - 0.00012\ (Q\Delta T_{PL})$	2.16	$F_{1,63}$= 51.3	7.06	0.67	1.61
	$X = 10.16 - 2.64\ \ln\ (\Delta T_{PL})$	2.16	$F_{1,63}$= 98.4	7.06	0.78	1.36
	$X = 26.88 - 2.30\ \ln\ (Q\Delta T_{PL})$	2.16	$F_{1,63}$= 73.38	7.06	0.73	1.48
	$X = 10.73 - 2.03\ \sqrt{\Delta T}_{PL}$	2.16	$F_{1,63}$=106.77	7.06	0.79	1.32
	$X = 9.61 - 0.04\ \sqrt{Q\Delta T}_{PL}$	2.16	$F_{1,63}$= 67.78	7.06	0.72	1.51

Notes:
\# X (Distance from outfall measured along plume centreline) in 10^3 ft.
* ΔT_{PL} (Temperature difference between plume and lake) in °F
\#* Q (Discharge from outfall) in cfs.
1 'Summer' is defined when lake temperature ≥45°F
2 'Winter' is defined when lake temperature <45°F

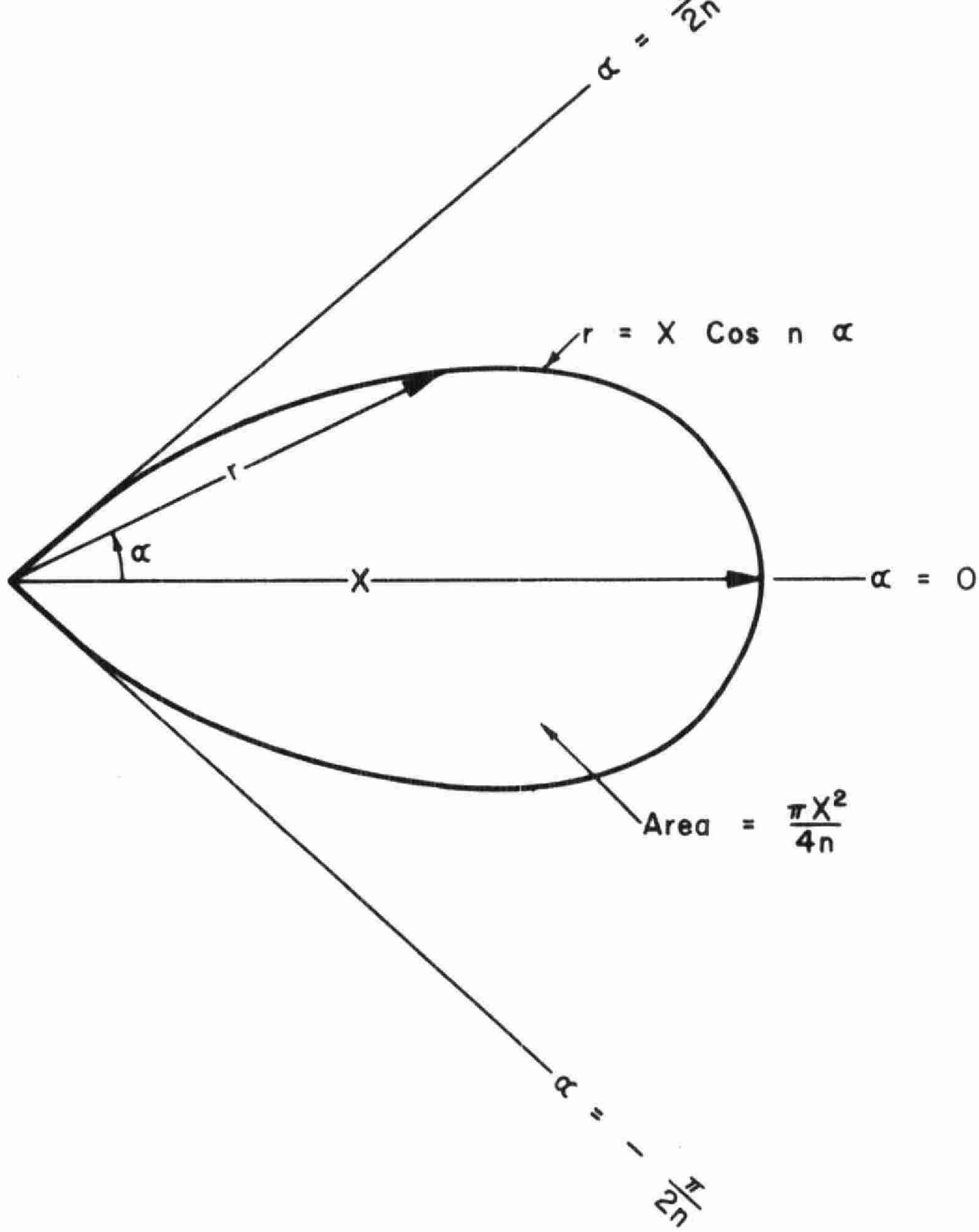
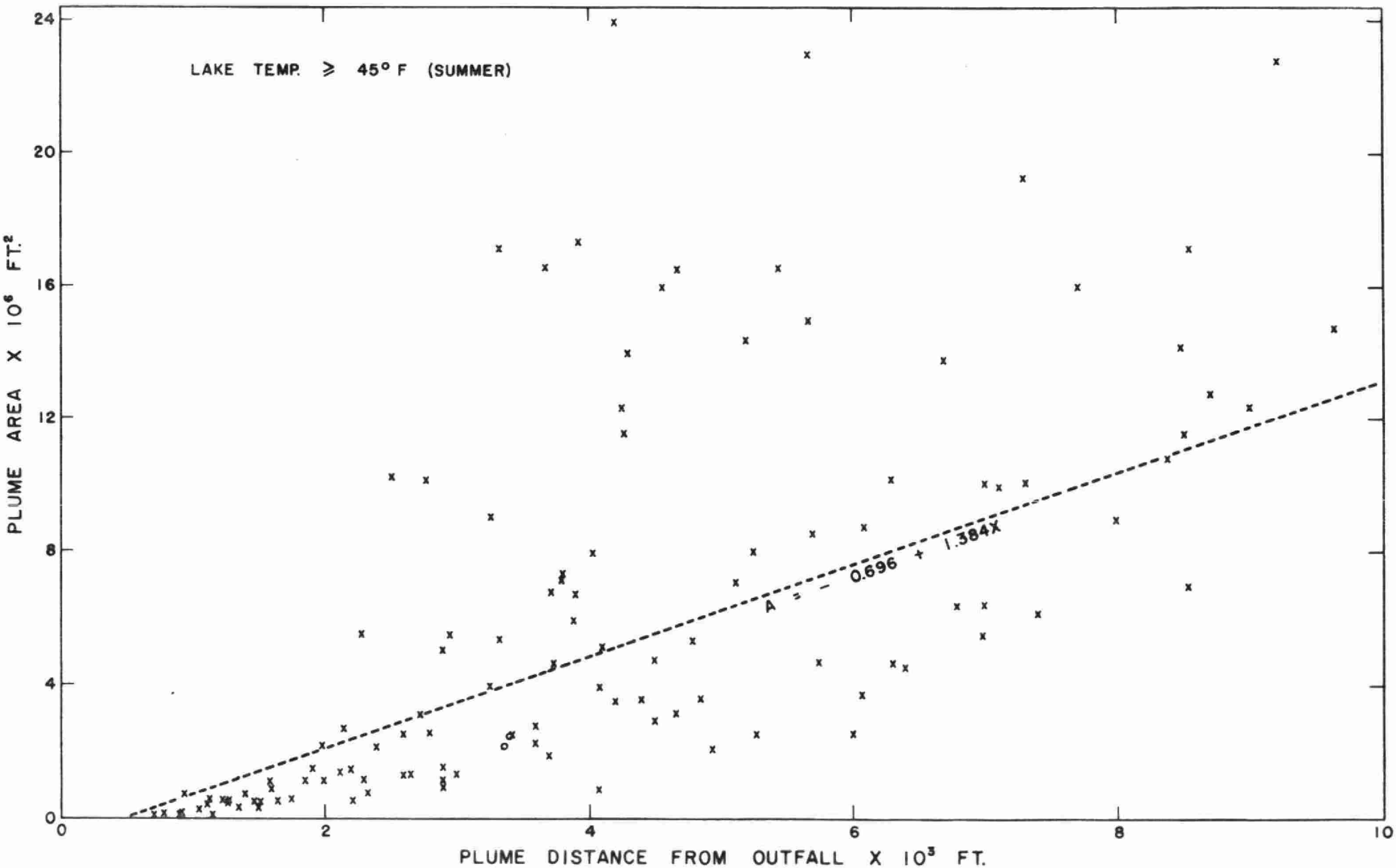
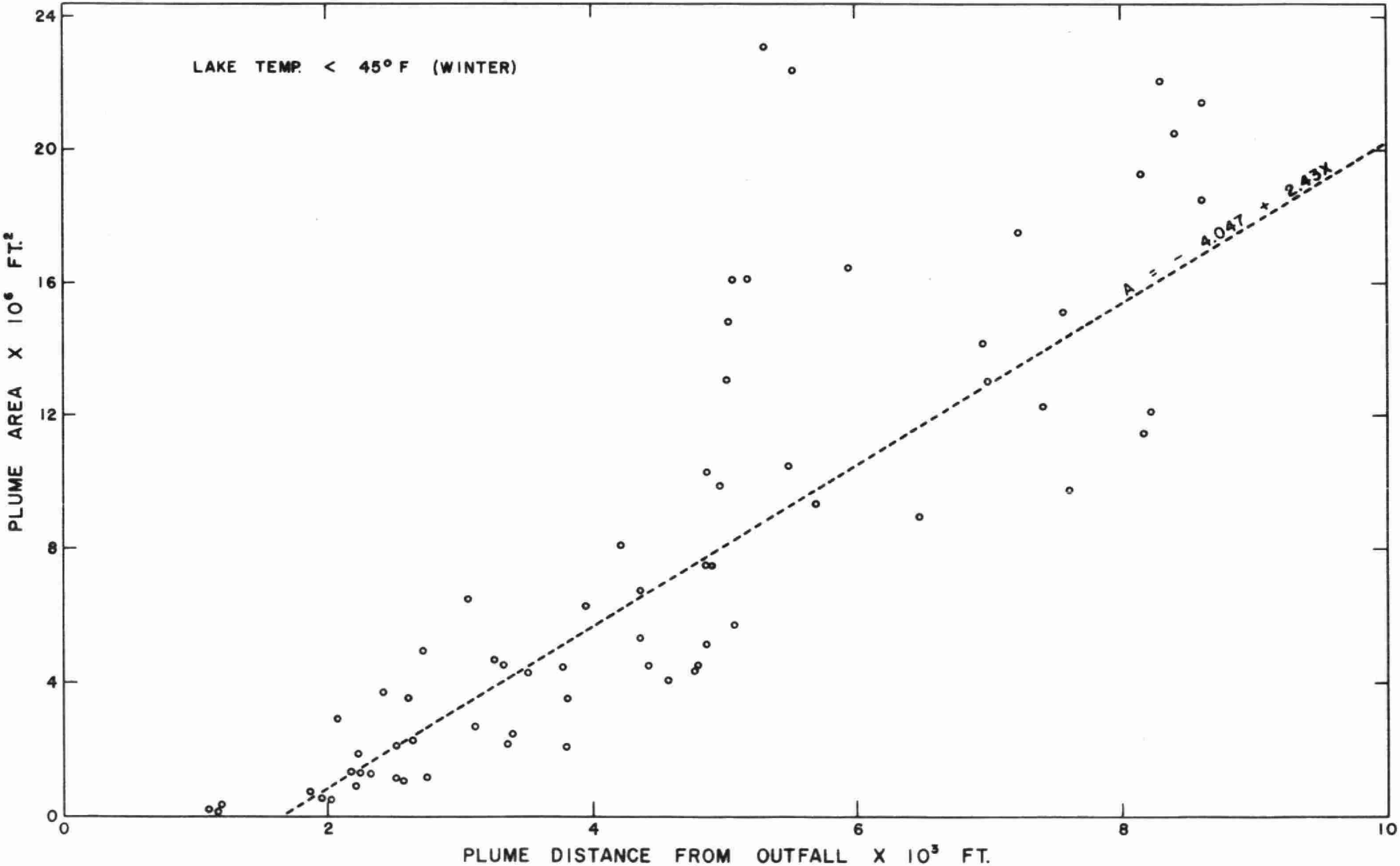


FIGURE 1
ISOTHERM SHAPE



LAKE TEMP. ≥ 45° F (SUMMER)
A = − 0.696 + 1.384X
PLUME AREA X 10⁶ FT.²
PLUME DISTANCE FROM OUTFALL X 10³ FT.
0
4
8
12
16
20
24
0
2
4
6
8
10

FIGURE 2



LAKE TEMP. < 45° F (WINTER)
A = - 4.047 + 2.43X
PLUME AREA X 10⁶ FT.²
PLUME DISTANCE FROM OUTFALL X 10³ FT.
0
4
8
12
16
20
24
0
2
4
6
8
10

FIGURE 3

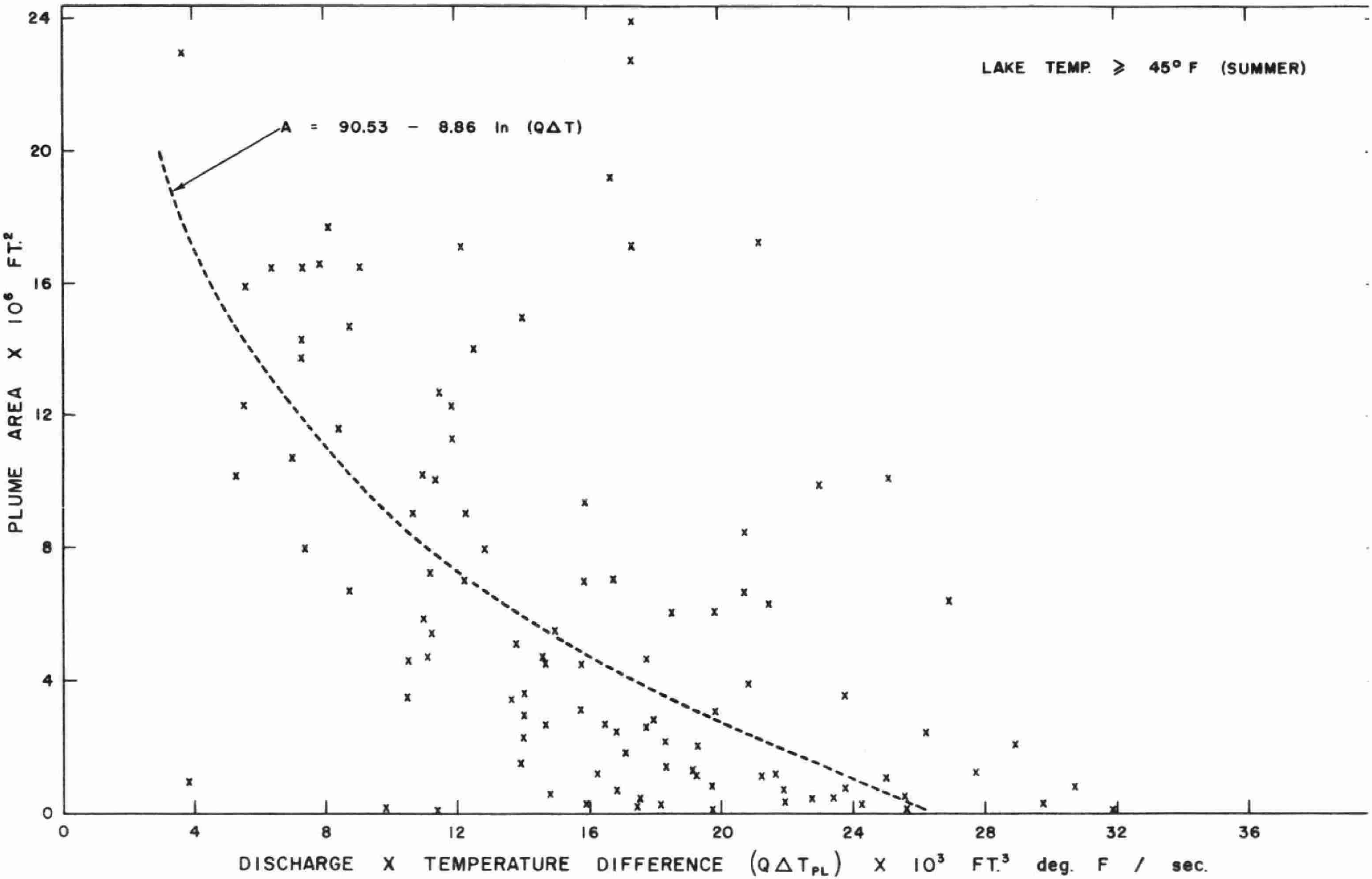
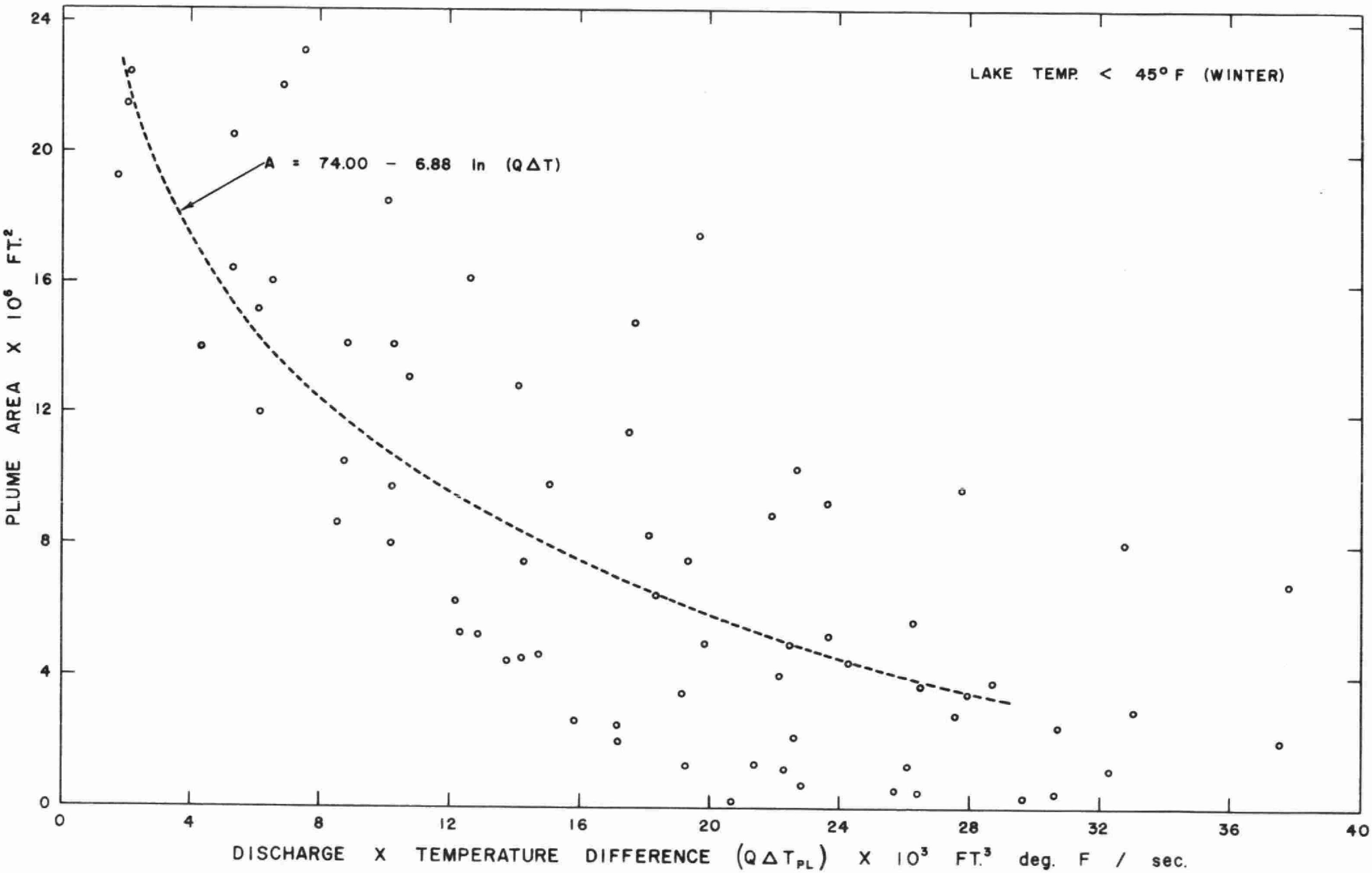

LAKE TEMP. ≥ 45° F (SUMMER)
A = 90.53 − 8.86 ln (QΔT)
PLUME AREA X 10⁶ FT.²
DISCHARGE X TEMPERATURE DIFFERENCE (QΔT_PL) X 10³ FT.³ deg. F / sec.
0
4
8
12
16
20
24
28
32
36

FIGURE 4



LAKE TEMP. < 45° F (WINTER)
A = 74.00 − 6.88 ln (Q ΔT)
PLUME AREA X 10⁶ FT.²
DISCHARGE X TEMPERATURE DIFFERENCE (Q Δ T PL) X 10³ FT.³ deg. F / sec.
0
4
8
12
16
20
24
28
32
36
40

FIGURE 5

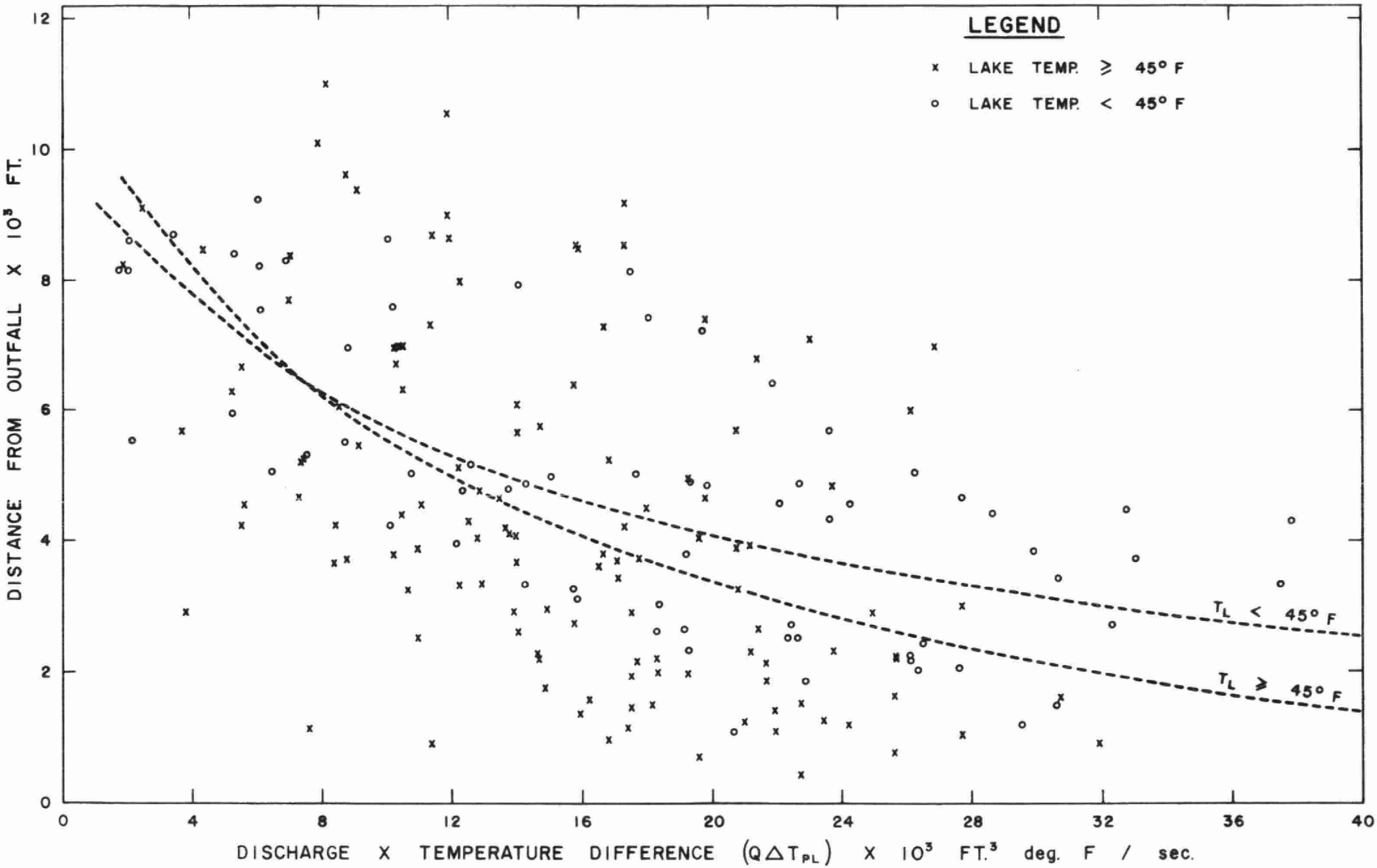

LEGEND
x LAKE TEMP. ≥ 45° F
o LAKE TEMP. < 45° F
DISTANCE FROM OUTFALL X 10³ FT.
DISCHARGE X TEMPERATURE DIFFERENCE (Q△T_PL) X 10³ FT.³ deg. F / sec.
T_L < 45° F
T_L ≥ 45° F
0
2
4
6
8
10
12
0
4
8
12
16
20
24
28
32
36
40

FIGURE 6

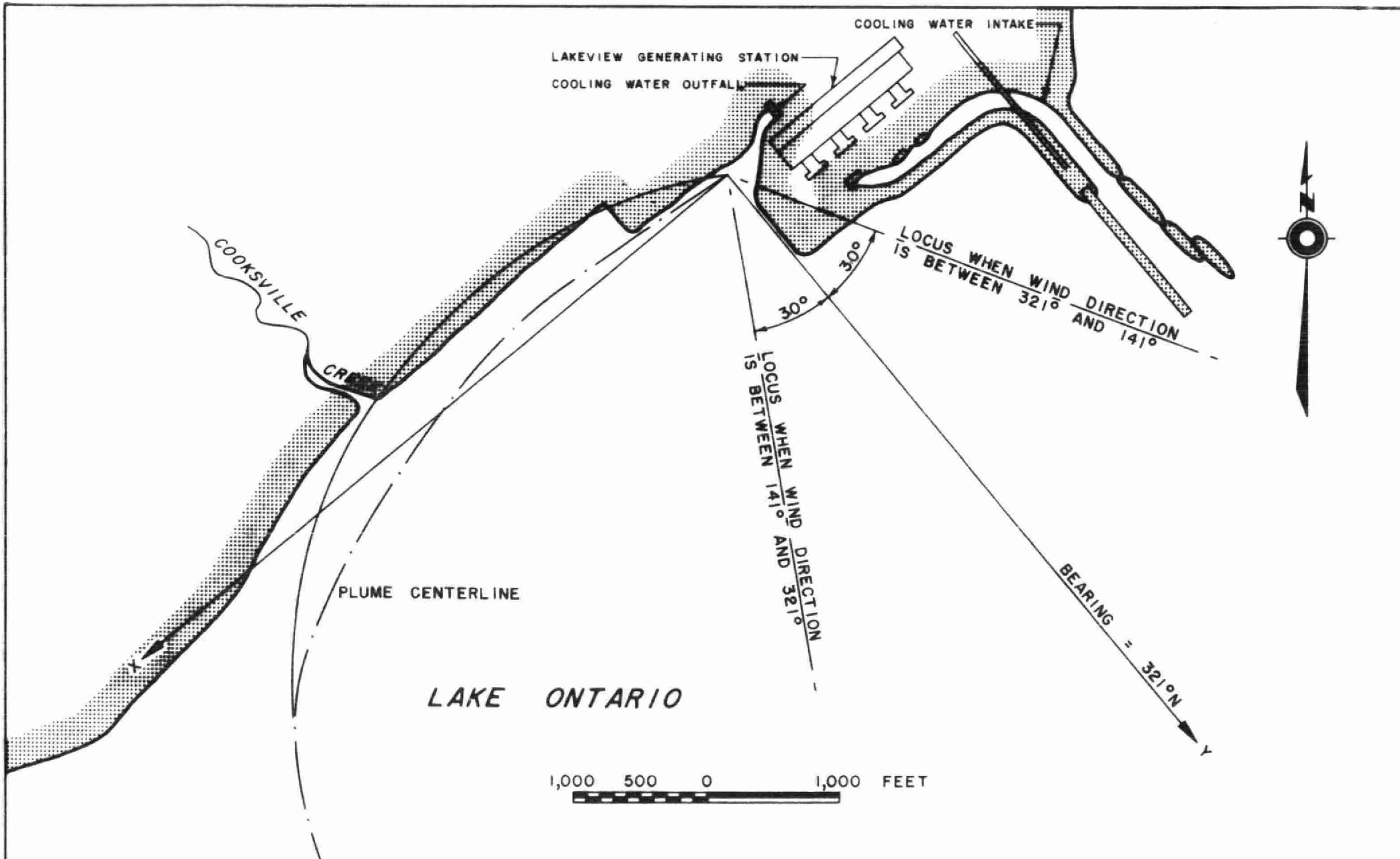


FIGURE 7

THERMAL PLUME CENTERLINE LOCII

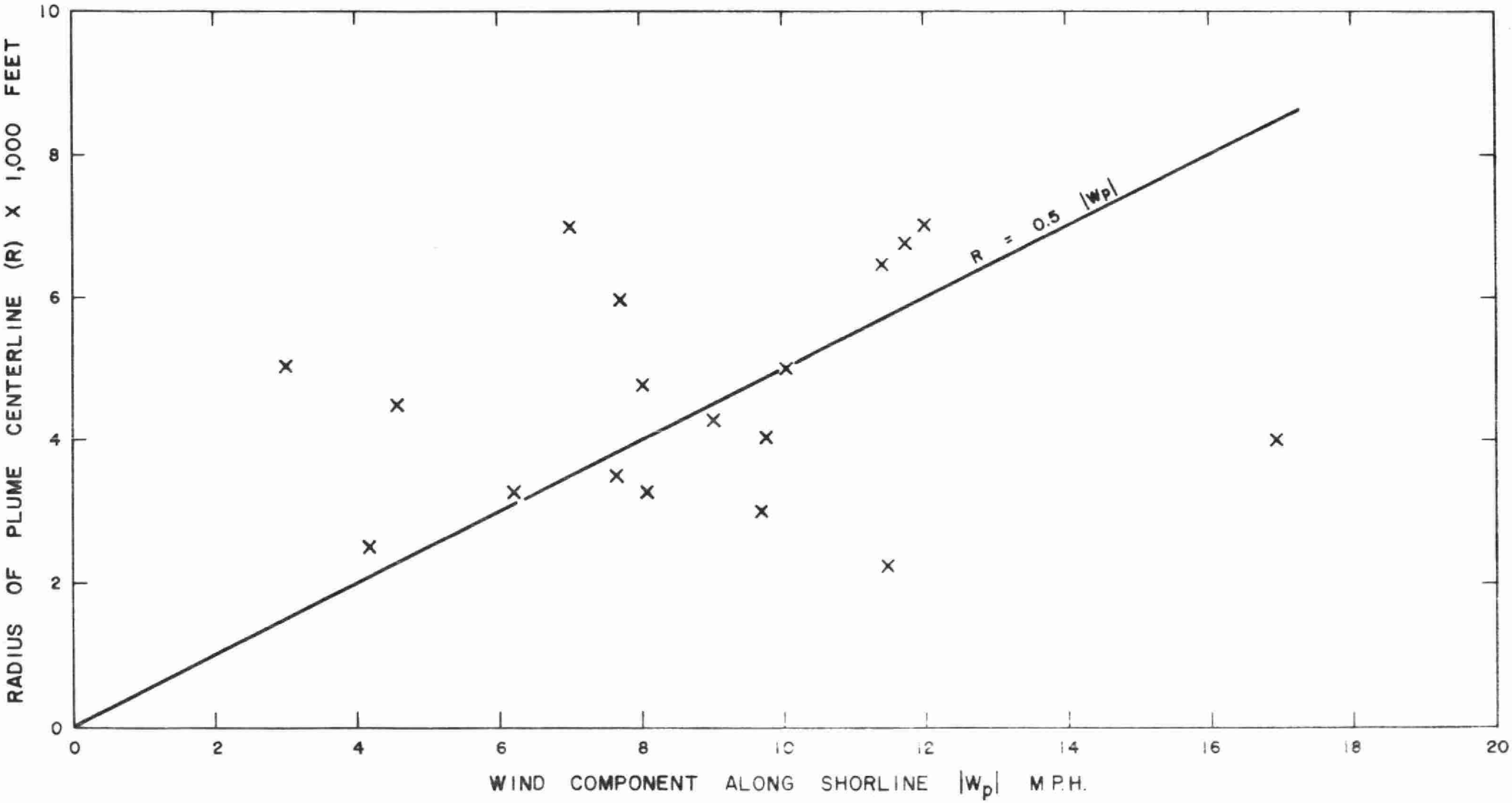

RADIUS OF PLUME CENTERLINE (R) X 1,000 FEET
0
2
4
6
8
10
R = 0.5 |Wp|
0
2
4
6
8
10
12
14
16
18
20
WIND COMPONENT ALONG SHORLINE $|W_p|$ M.P.H.

FIGURE 8

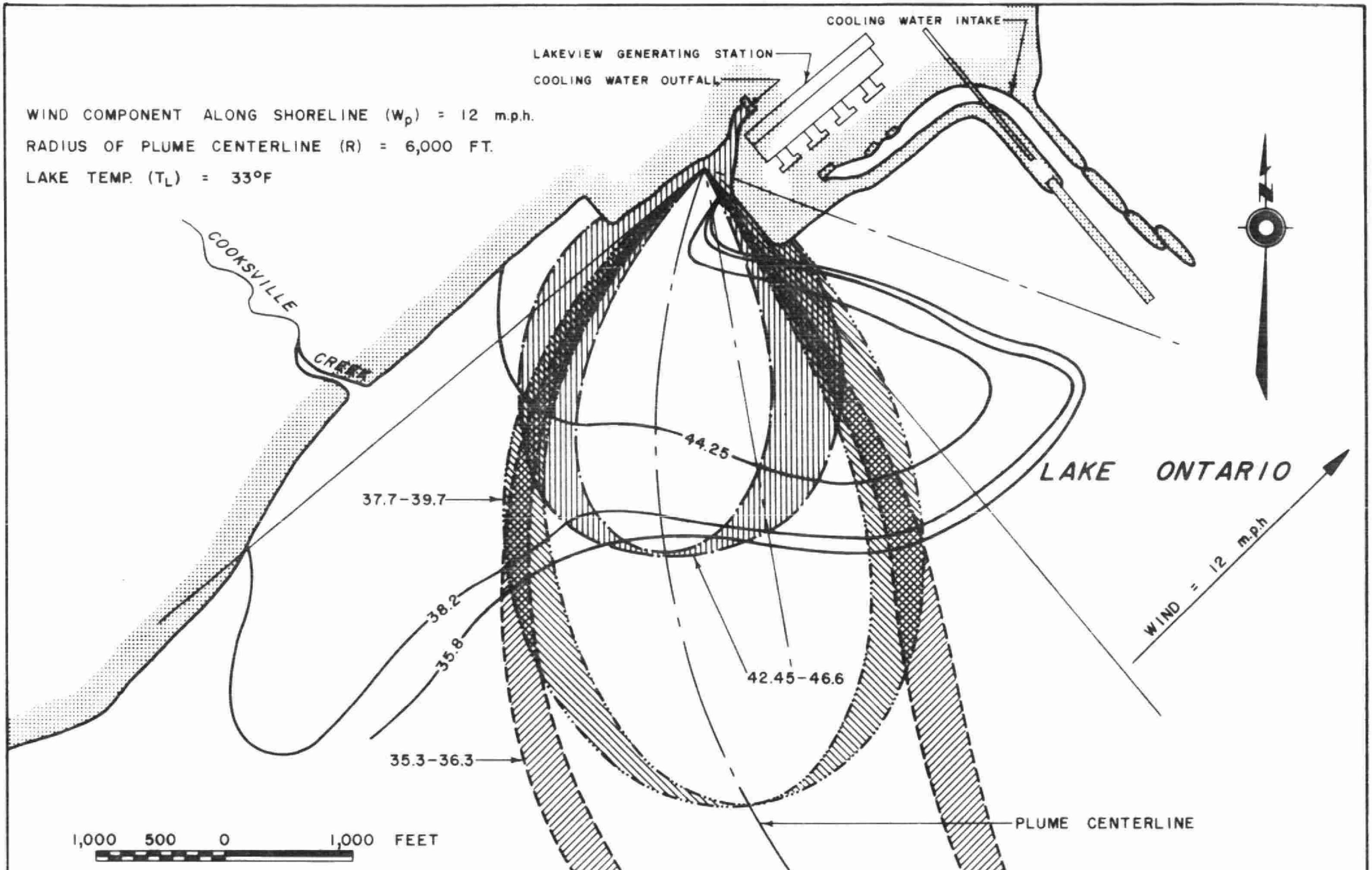


FIGURE 9
OBSERVED AND PREDICTED ISOTHERMS (FEB. 4, 1970)

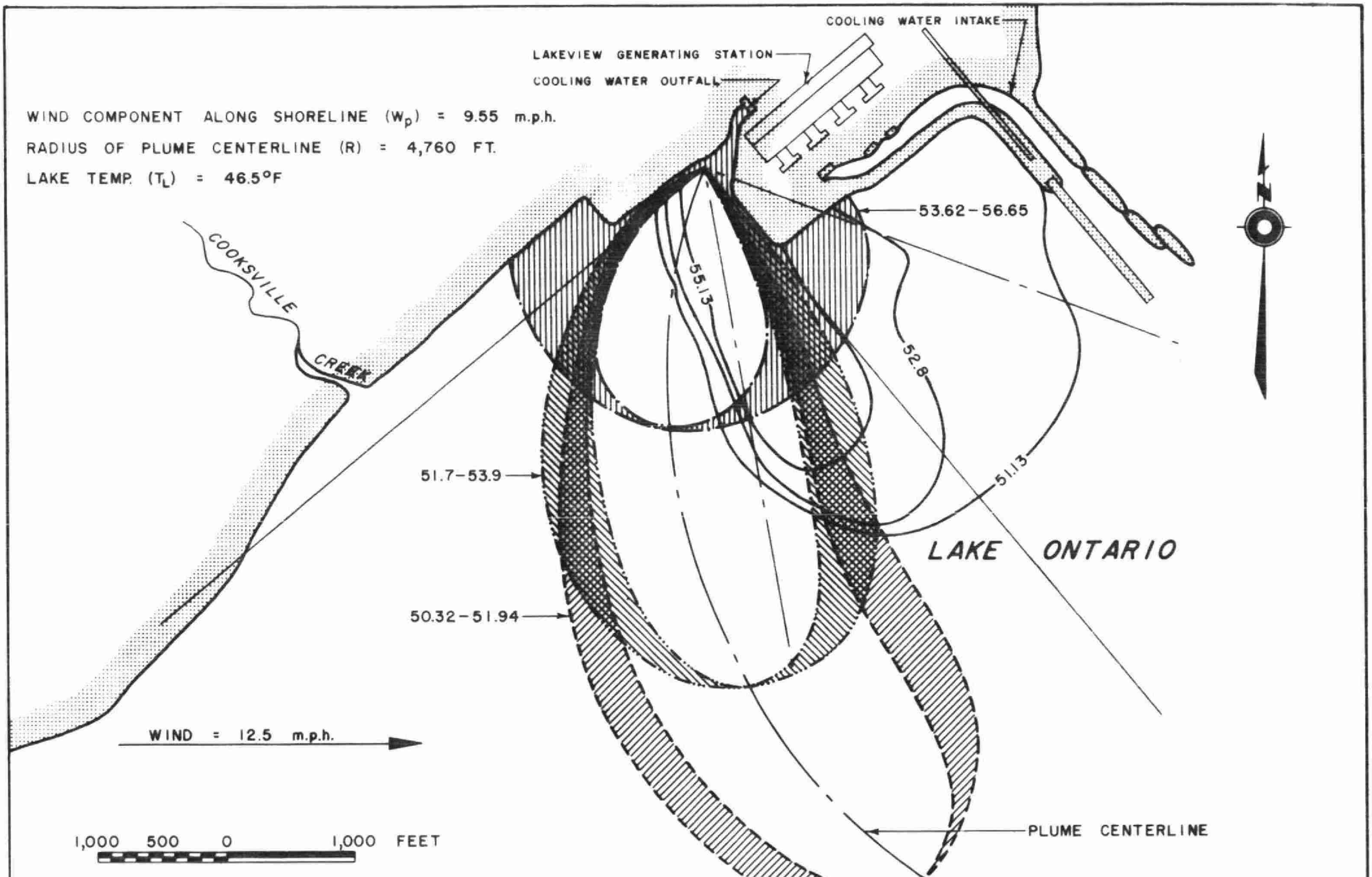


FIGURE 10
OBSERVED AND PREDICTED ISOTHERMS (NOV. 13, 1969)

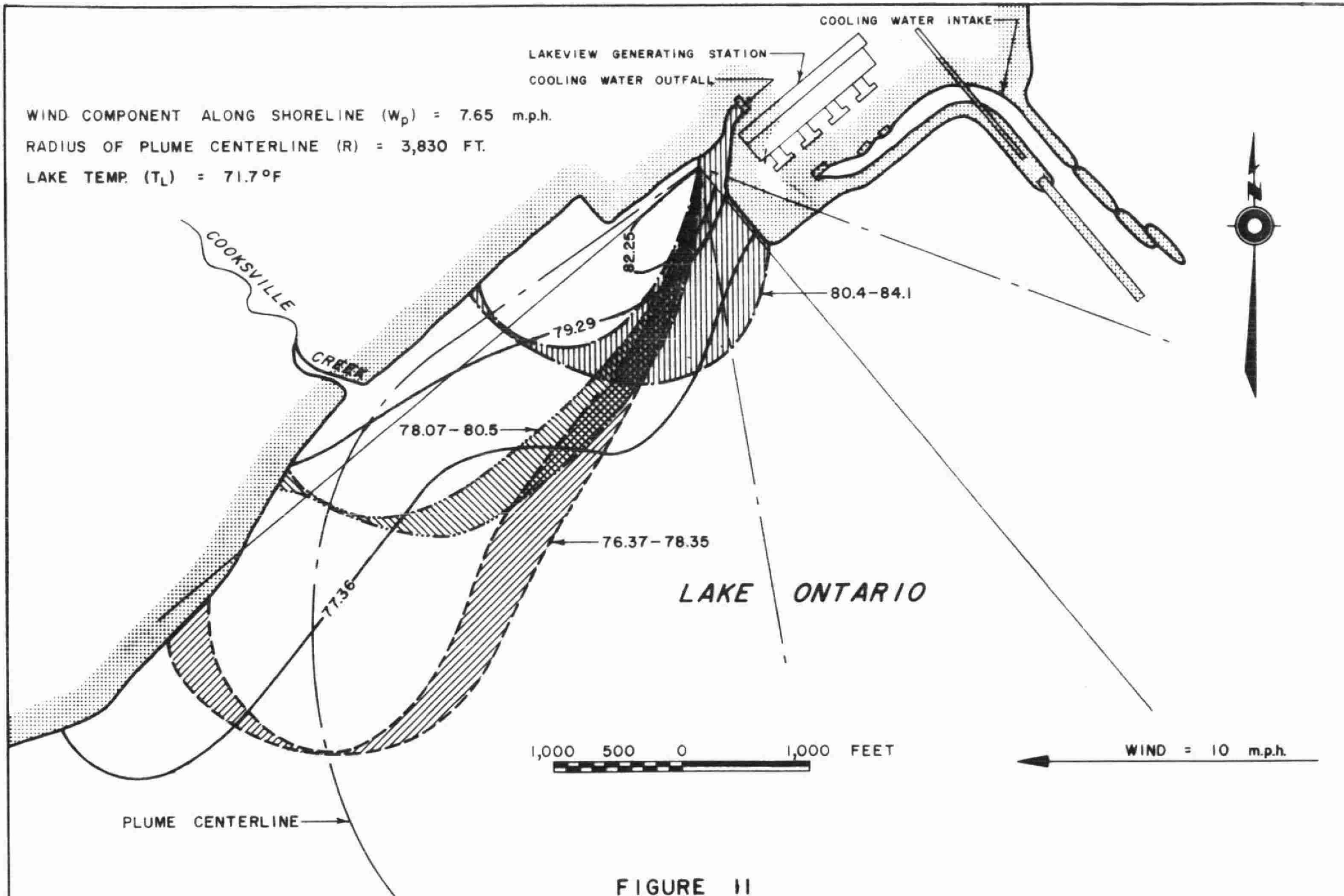


FIGURE 11
OBSERVED AND PREDICTED ISOTHERMS (JULY 23, 1969)

APPENDIX I

Sample Computations for Thermal Plume Predictions July 23, 1969.

Relevant Data

Air temperature, $T_A = 75°F$

Lake water temperature, $T_L = 71.7°F$

Wind Speed, W = 10 mph @ 271°

Outfall Discharge, Q = 1750 cfs

Definition of Plume Centreline

Since the shoreline is @ 231°, the angle Θ between shoreline and prevailing wind is:

$\Theta = 271-231 = 40°$

$\therefore$ Wind Component along shoreline

$W_P = W \cos\Theta = 10 \cos 40 = 7.65$ mph

But $R = 500|W_p|$ ft

$\therefore$ R = 3830 ft.

The arc of the plume centreline is drawn with a radius of 3830 ft. on locii of centres (see Figure 4). Since the arc does not lie wholly on the water, the plume centreline is smoothly joined to outfall, avoiding the shore.

Definition of Plume Area

Since the lake temperature is 71.7°F, the computations are done for 'summer' regime. The prediction equation is accordingly selected from Table 1.

$A = -0.696 + 1.384X$ ....(1)

where A = area in 10^6 ft.^{2}

X = distance from outfall, measured along the plume centreline in 10^{-3} ft.

The following computations are for X = 4 (4000 ft.)

(a) Mean Area

$A = -0.696 + 1.384(4) = 4.84$

(b) Maximum Area (One standard error)

Standard error of equation (1) = 3.11 from Table 1.

$\therefore$ Maximum area $A^* = 4.84 + 3.11 = 7.95$

Determination of Temperature Difference (ΔT_{PL})

The prediction equation is selected from Table 3 as follows:

$$A = 90.53 - 8.86 \ln (Q\Delta T_{PL}) \quad (2)$$

where $\Delta T_{PL} = T_P - T_L$ in °F

T_P = plume temperature in °F

(a) ΔT_{PL} (Mean Area)

Substituting the value A = 4.84, calculated above for X = 4 and Q = 1750 in equation (2), ΔT_{PL} is computed.

$\therefore \Delta T_{PL} = 9.05$

$\therefore T_P = T_L + \Delta T_{PL} = 71.7 + 9.05 = 80.75°F$

(b) ΔT_{PL}^* (Maximum Area)

Using $A^* = 7.95$ and Q = 1750 in equation (2), ΔT_{PL}^* is computed.

$\Delta T_{PL}^* = 6.37°F$

$\therefore T_P^* = T_L + \Delta T_{PL}^* = 71.7 + 6.37 = 78.07°F$

Determination of Isothermal Contour

Equation (3) describes a multi-leaved rose in polar co-ordinates (r, α)

$$r = x \operatorname{Cos} n\alpha \quad (3)$$

where n governs the number of leaves and therefore the lateral spread. (See Figure 1).

Area enclosed by the curve may be shown as:

$$A = \frac{\pi x^2}{4n} \quad (4)$$

For X = 4, substituting the values of A = 4.84 and $A^* = 7.95$ in equation (3) values of n and n* are computed

n = 2.6

n*= 1.58

Substituting these values of n and n* in equation (3), the corresponding curves can be plotted.

APPENDIX II

Data Used

Survey Date	Discharge Flow	Outfall Temp.	Lake Temp.	Air Temp.
	cfs	^oF	^oF	^oF
April 4, 1969	1970	56.2	40.0	52.0
May 22, 1969	1400	62.0	46.0	50.0
May 23, 1969	1400	61.0	46.0	57.0
June 19, 1969	1830	58.6	47.0	60.0
July 15, 1969	1730	76.0	61.7	82.0
July 16, 1969	1730	78.0	62.2	83.0
July 17, 1969	1620	74.0	61.7	83.0
July 22, 1969	1750	84.0	69.3	75.0
July 23, 1969	1750	84.0	71.7	75.0
July 24, 1969	1750	86.0	71.5	76.0
Sept. 9, 1969	1970	72.0	59.0	66.0
Sept. 10, 1969	1750	70.0	58.0	69.0
Sept. 11, 1969	1750	74.0	59.0	63.0
Sept. 30, 1969	1830	73.0	58.0	66.0
Oct. 1, 1969	1830	73.0	58.0	60.0
Oct. 2, 1969	1980	75.0	59.5	61.0
Oct. 21, 1969	1740	62.0	43.0	48.0
Oct. 22, 1969	1760	58.0	43.0	31.0
Oct. 23, 1969	1720	56.0	42.0	33.0
Nov. 12, 1969	2130	64.0	47.0	47.5
Nov. 13, 1969	2140	60.0	46.5	42.5
Nov. 25, 1969	2040	57.0	41.0	45.0

APPENDIX II (Cont'd)

Data Used

Survey Date	Discharge Flow	Outfall Temp.	Lake Temp.	Air Temp.
	cfs	^{o}F	^{o}F	^{o}F
Nov. 26, 1969	2030	56.0	41.0	40.0
Nov. 27, 1969	2010	56.0	41.0	32.5
Jan. 28, 1970	2520	56.0	35.0	36.0
Feb. 3, 1970	2190	51.7	34.0	8.0
Feb. 4, 1970	2150	51.4	33.0	13.0
Feb. 20, 1970	2205	61.0	35.0	22.0
May 20, 1970	1924	63.0	45.0	52.0
May 21, 1970	1920	64.0	46.0	55.0
June 10, 1970	1860	66.0	61.0	71.0
June 11, 1970	1850	66.0	54.0	71.0
June 11, 1970	1740	65.0	54.0	84.0
Sept. 1, 1970	2270	65.0	56.0	65.0
Sept. 2, 1970	2440	68.0	59.0	60.0
Sept. 2, 1970	2380	67.0	55.0	65.0
Oct. 20, 1970	2080	70.0	54.0	53.0
Oct. 20, 1970	1980	72.0	54.0	54.0
Oct. 22, 1970	1970	72.0	55.0	55.0
Oct. 22, 1970	1630	72.0	55.0	56.0